Remainders of Security: from Modular Arithmetic to Cryptography

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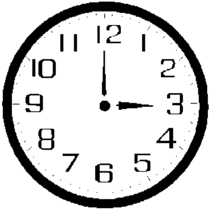
Newnham College

6 July 2017, Open Day

Remainders

- If it is 3 o'clock now, what time is it in 10 hours?
- If it is Thursday now, what day is it in 9 days?
- If it is summer now, what season will it be in 100 seasons?
- If it is midday now, will it be light or dark in 539 hours?



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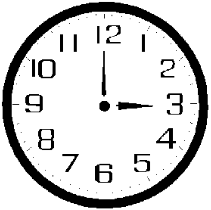

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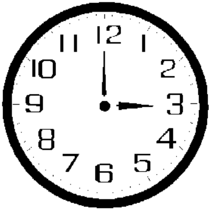

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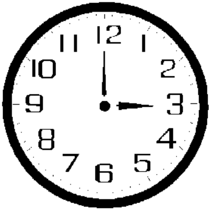

Calculating with remainders

Writing the above answers mathematically

- $3 + 10 \equiv 1 \pmod{12}$ So 1 o'clock.
- $4 + 9 \equiv 6 \pmod{7}$ So Saturday.
- $2 + 100 \equiv 2 \pmod{4}$ So summer again.
- $12 + 539 \equiv 12 + 480 + 59 \equiv 12 + 11 \equiv 23 \pmod{24}$
  So it will be 23h, or 11pm, so dark.

Modular Arithmetic

Formally

For whole numbers x, y and n we write

$$x \equiv y \pmod{n} \iff (x-y) = kn \quad \text{for some whole number } k.$$

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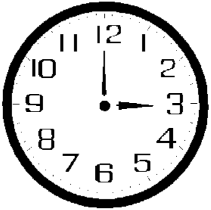

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Multiplication modulo 5

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Inverse

We say y is an inverse of x mod n if $xy \equiv 1 \pmod{n}$.

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Lemma

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For n and a coprime, consider the numbers $a, 2a, 3a, \ldots, (n-1)a \bmod n$.

- Is it possible that any of these is $0 \pmod{n}$?

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Theorem (Little Fermat)

If p prime and a not a multiple of p, then

$$a^{p-1} \equiv 1 \pmod{p}$$

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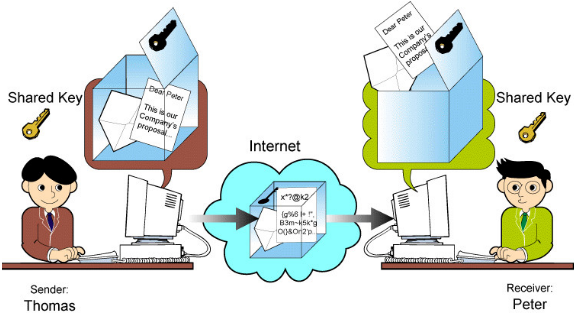
Cryptography



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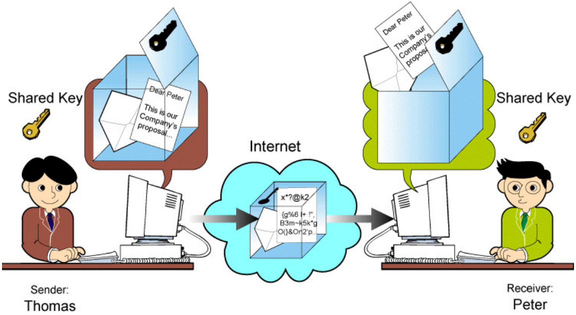

- write secret messages
- store data securely
- secure internet payment
- secret radio transmission in war
- ...

Caesar Cipher

How does it work?

- Our friend moves to Australia, we want to send them a secret letter.
- We can use different “shifts”: our key.
- We write secret sentence using key.
- How will recipient know key?

Link to modular arithmetic

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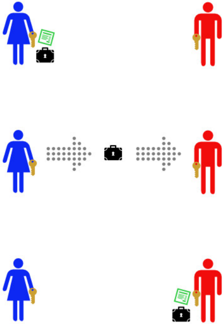
- To decipher, recipient needs to calculate
$$\beta - \varphi \pmod{26}$$
to get your original message α back.

Symmetric Key Cryptography

Problems

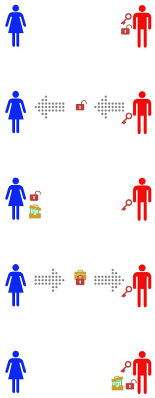
- Alice and Bob want secret communication.
- Both need same key.
- Problem: safe key exchange.
- Doesn't work for internet shopping.



Public Key Cryptography

Padlock metaphor

- Bob has padlock and matching key.
- Alice can get open padlock from internet.
- Alice padlocks the message for Bob.
- Message now safe to send.
- Only Bob has the key to open it.



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- Decrypt ciphertext y as $x \equiv y^d \pmod{n}$.

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$$x^{ed} = x^{k\varphi+1} = x \cdot x^{k(p-1)(q-1)} = x \cdot (x^{(p-1)})^{k(q-1)}.$$

But Little Fermat $\Rightarrow$ $x^{(p-1)} \equiv 1 \pmod{p}$ as long as $x \not\equiv 0 \pmod{p}$.

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Does it really work?

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Hurray!

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Multiplying vs. Factorising

- Calculate $23 \cdot 37$.
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- Which was faster/easier?
- To decipher, need to know d, for which we need φ, for which we need p and q: hard to get.

I hope you had some fun!