The fundamental group functor as a Kan extension

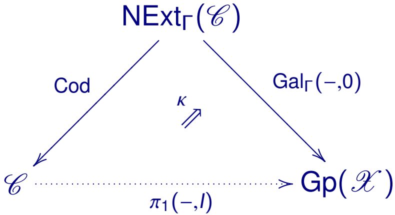
Julia Goedecke

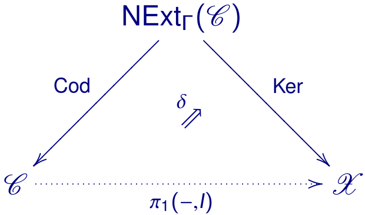
University of Cambridge

joint work with Tomas Everaert and Tim Van der Linden

7 - 13 July 2013, CT13 Sydney

Aim of the talk





Why?

- In topological example gives some universal properties of the "usual" fundamental group, and also the connecting homomorphism in exact sequence induced by a fibration.
- In algebraic examples gives another approach to semi-abelian homology, and again some information about a connecting homomorphism.

Why?

- In topological example gives some universal properties of the "usual" fundamental group, and also the connecting homomorphism in exact sequence induced by a fibration.
- In algebraic examples gives another approach to semi-abelian homology, and again some information about a connecting homomorphism.

Galois structures

Definition (Janelidze)

A Galois structure Γ consists of an adjunction

$$\mathscr{C} \underset{H}{\overset{I}{\rightleftarrows}} \mathscr{X} \quad (I \dashv H)$$

with unit η and counit ϵ, as well as classes of maps $\mathscr{E}$ in $\mathscr{C}$ and $\mathscr{F}$ in $\mathscr{X}$ satisfying certain axioms.

Examples of Galois structures

- Groups with subcategory abelian groups, regular epis.
- Semi-abelian $\mathscr{C}$ with Birkhoff subcategory, regular epis.
- Locally connected topological spaces and sets.
  H is discrete topology functor,
  $I = \pi_0$, connected components functor.
  $\mathscr{E}$ = local homeomorphisms (étale maps), $\mathscr{F}$ = all maps.
- Opposite of finite dimensional k-algebras, finite sets, each with all maps. The adjunction is defined through idempotent decomposition of k-algebras: a k-algebra is sent to its set of primitive idempotents.

Examples of Galois structures

- Groups with subcategory abelian groups, regular epis.
- Semi-abelian $\mathscr{C}$ with Birkhoff subcategory, regular epis.
- Locally connected topological spaces and sets.
  H is discrete topology functor,
  $I = \pi_0$, connected components functor.
  $\mathscr{E}$ = local homeomorphisms (étale maps), $\mathscr{F}$ = all maps.
- Opposite of finite dimensional k-algebras, finite sets, each with all maps. The adjunction is defined through idempotent decomposition of k-algebras: a k-algebra is sent to its set of primitive idempotents.

Examples of Galois structures

- Groups with subcategory abelian groups, regular epis.
- Semi-abelian $\mathscr{C}$ with Birkhoff subcategory, regular epis.
- Locally connected topological spaces and sets.
  H is discrete topology functor,
  $I = \pi_0$, connected components functor.
  $\mathscr{E}$ = local homeomorphisms (étale maps), $\mathscr{F}$ = all maps.
- Opposite of finite dimensional k-algebras, finite sets, each with all maps. The adjunction is defined through idempotent decomposition of k-algebras: a k-algebra is sent to its set of primitive idempotents.

Examples of Galois structures

- Groups with subcategory abelian groups, regular epis.
- Semi-abelian $\mathscr{C}$ with Birkhoff subcategory, regular epis.
- Locally connected topological spaces and sets.
  H is discrete topology functor,
  $I = \pi_0$, connected components functor.
  $\mathscr{E} =$ local homeomorphisms (étale maps), $\mathscr{F} =$ all maps.
- Opposite of finite dimensional k-algebras, finite sets, each with all maps. The adjunction is defined through idempotent decomposition of k-algebras: a k-algebra is sent to its set of primitive idempotents.

Special maps in Galois structures

- trivial coverings:

$$\begin{array}{ccc} A & \xrightarrow{\eta_A} & HIA \\ {\scriptstyle \mathscr{E} \ni f}\downarrow & \lrcorner & \downarrow{\scriptstyle HIf} \\ B & \xrightarrow[\eta_B]{} & HIB \end{array}$$

(*cartesian wrt. I*)

- monadic extensions: $p\colon E \longrightarrow B$ in $\mathscr{E}$ with $p^*\colon (\mathscr{E} \downarrow B) \longrightarrow (\mathscr{E} \downarrow E)$ monadic. (*good to pull back along*)
- coverings (or central extensions): $f \in \mathscr{E}$ with $p^*(f)$ trivial for some monadic p. (*locally trivial*)
- normal extensions: monadic p with trivial kernel pair projections.

Special maps in Galois structures

- trivial coverings:

$$\begin{array}{ccc} A & \xrightarrow{\eta_A} & HIA \\ {\scriptstyle \mathscr{E} \ni f}\downarrow & & \downarrow{\scriptstyle HIf} \\ B & \xrightarrow[\eta_B]{} & HIB \end{array}$$

  (*cartesian wrt. I*)

- monadic extensions: $p\colon E \longrightarrow B$ in $\mathscr{E}$ with $p^*\colon (\mathscr{E} \downarrow B) \longrightarrow (\mathscr{E} \downarrow E)$ monadic.
  (*good to pull back along*)
- coverings (or central extensions): $f \in \mathscr{E}$ with $p^*(f)$ trivial for some monadic p. (*locally trivial*)
- normal extensions: monadic p with trivial kernel pair projections.

Special maps in Galois structures

- trivial coverings:
$$\begin{array}{ccc} A & \xrightarrow{\eta_A} & HIA \\ {\mathscr{E}\ni f}\downarrow & \lrcorner & \downarrow{HIf} \\ B & \xrightarrow[\eta_B]{} & HIB \end{array}$$
(cartesian wrt. I)
- monadic extensions: $p\colon E \longrightarrow B$ in $\mathscr{E}$ with $p^*\colon (\mathscr{E} \downarrow B) \longrightarrow (\mathscr{E} \downarrow E)$ monadic. *(good to pull back along)*
- coverings (or central extensions): $f \in \mathscr{E}$ with $p^*(f)$ trivial for some monadic p. *(locally trivial)*
- normal extensions: monadic p with trivial kernel pair projections.

Special maps in Galois structures

- trivial coverings: $$\begin{array}{ccc} A & \xrightarrow{\eta_A} & HIA \\ {\scriptstyle \mathscr{E} \ni f}\downarrow & \lrcorner & \downarrow{\scriptstyle HIf} \\ B & \xrightarrow[\eta_B]{} & HIB \end{array}$$ *(cartesian wrt. I)*
- monadic extensions: $p \colon E \longrightarrow B$ in $\mathscr{E}$ with $p^* \colon (\mathscr{E} \downarrow B) \longrightarrow (\mathscr{E} \downarrow E)$ monadic.
  (good to pull back along)
- coverings (or central extensions): $f \in \mathscr{E}$ with $p^*(f)$ trivial for some monadic p. *(locally trivial)*
- normal extensions: monadic p with trivial kernel pair projections.

Examples

- Groups with abelian groups:
  - monadic extensions: all regular epis.
  - central extensions as usual, kernel inside centre;
- topological example:
  - monadic extensions: surjective étale maps;
  - coverings: usual topological sense;
  - normal extensions: regular coverings.
- In many algebraic examples, central = normal.

Examples

- Groups with abelian groups:
    - monadic extensions: all regular epis.
    - central extensions as usual, kernel inside centre;
- topological example:
    - monadic extensions: surjective étale maps;
    - coverings: usual topological sense;
    - normal extensions: regular coverings.
- In many algebraic examples, central = normal.

Examples

- Groups with abelian groups:
  - monadic extensions: all regular epis.
  - central extensions as usual, kernel inside centre;
- topological example:
  - monadic extensions: surjective étale maps;
  - coverings: usual topological sense;
  - normal extensions: regular coverings.
- In many algebraic examples, central = normal.

Admissibility

From now on H is inclusion.

- admissible Galois structure: I preserves pullbacks along trivial coverings.
- When $\mathscr{E}$ is all maps,
  admissible = semi-left exact = Street fibration.
- $\Rightarrow$ Trivial coverings are pullback-stable.
- Think "trivial coverings are pullback-closure of $\mathscr{F}$ in $\mathscr{C}$".
- If monadic extensions pullback-stable, then also normal extensions and coverings pullback-stable.

Admissibility

From now on H is inclusion.

- admissible Galois structure: I preserves pullbacks along trivial coverings.
- When $\mathscr{E}$ is all maps,
  admissible = semi-left exact = Street fibration.
- $\Rightarrow$ Trivial coverings are pullback-stable.
- Think "trivial coverings are pullback-closure of $\mathscr{F}$ in $\mathscr{C}$".
- If monadic extensions pullback-stable, then also normal extensions and coverings pullback-stable.

Admissibility

From now on H is inclusion.

- admissible Galois structure: I preserves pullbacks along trivial coverings.
- When $\mathscr{E}$ is all maps,
  admissible = semi-left exact = Street fibration.
- $\Rightarrow$ Trivial coverings are pullback-stable.
- Think "trivial coverings are pullback-closure of $\mathscr{F}$ in $\mathscr{C}$".
- If monadic extensions pullback-stable, then also normal extensions and coverings pullback-stable.

Admissibility

From now on H is inclusion.

- admissible Galois structure: I preserves pullbacks along trivial coverings.
- When $\mathscr{E}$ is all maps,
  admissible = semi-left exact = Street fibration.
- $\Rightarrow$ Trivial coverings are pullback-stable.
- Think "trivial coverings are pullback-closure of $\mathscr{F}$ in $\mathscr{C}$".
- If monadic extensions pullback-stable, then also normal extensions and coverings pullback-stable.

Admissibility

From now on H is inclusion.

- admissible Galois structure: I preserves pullbacks along trivial coverings.
- When $\mathscr{E}$ is all maps,
  admissible = semi-left exact = Street fibration.
- $\Rightarrow$ Trivial coverings are pullback-stable.
- Think "trivial coverings are pullback-closure of $\mathscr{F}$ in $\mathscr{C}$".
- If monadic extensions pullback-stable, then also normal extensions and coverings pullback-stable.

Galois groupoid (Janelidze)

Take $\mathscr{C}$ pointed and $p\colon E \longrightarrow B$ normal extension.

- Galois groupoid $\mathrm{Gal}_\Gamma(p) = I\mathrm{Eq}(p)$

$$\begin{array}{ccccc} \mathrm{Eq}(p) & \overset{d}{\underset{c}{\rightrightarrows}} & E & \xrightarrow{p} & B \\ \eta \downarrow & & \downarrow \eta_E & & \\ I\mathrm{Eq}(p) & \overset{Id}{\underset{Ic}{\rightrightarrows}} & IE & & \end{array}$$

- Still groupoid, as I preserves defining pullbacks, because d and c trivial.
- Galois group $\mathrm{Gal}_\Gamma(p, 0) = \mathrm{Ker}\, Id \cap \mathrm{Ker}\, Ic$
  automorphisms at 0

Galois groupoid (Janelidze)

Take $\mathscr{C}$ pointed and $p\colon E \longrightarrow B$ normal extension.

- Galois groupoid $\mathrm{Gal}_\Gamma(p) = I\mathrm{Eq}(p)$

$$\begin{array}{ccccc} \mathrm{Eq}(p) & \overset{d}{\underset{c}{\rightrightarrows}} & E & \xrightarrow{\;p\;} & B \\ \downarrow{\scriptstyle \eta} & & \downarrow{\scriptstyle \eta_E} & & \\ I\mathrm{Eq}(p) & \overset{Id}{\underset{Ic}{\rightrightarrows}} & IE & & \end{array}$$

- Still groupoid, as I preserves defining pullbacks, because d and c trivial.
- Galois group $\mathrm{Gal}_\Gamma(p, 0) = \mathrm{Ker}\, Id \cap \mathrm{Ker}\, Ic$
  automorphisms at 0

Galois groupoid (Janelidze)

Take $\mathscr{C}$ pointed and $p\colon E \longrightarrow B$ normal extension.

- Galois groupoid $\mathrm{Gal}_\Gamma(p) = I\mathrm{Eq}(p)$

$$
\begin{array}{ccccc}
\mathrm{Eq}(p) & \overset{d}{\underset{c}{\rightrightarrows}} & E & \xrightarrow{p} & B \\
\downarrow{\scriptstyle \eta} & & \downarrow{\scriptstyle \eta_E} & & \\
I\mathrm{Eq}(p) & \overset{Id}{\underset{Ic}{\rightrightarrows}} & IE & &
\end{array}
$$

- Still groupoid, as I preserves defining pullbacks, because d and c trivial.
- Galois group $\mathrm{Gal}_\Gamma(p, 0) = \mathrm{Ker}\, Id \cap \mathrm{Ker}\, Ic$
  automorphisms at 0

Properties of Galois group functor

Morphisms in $\mathrm{NExt}_\Gamma(\mathscr{C})$

$$\begin{array}{ccc} E & \overset{f}{\underset{g}{\rightrightarrows}} & E' \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle p'} \\ B & \underset{b}{\longrightarrow} & B' \end{array}$$

induce

- homotopy on kernel pairs and Galois groupoids,
- same morphism $\mathrm{Gal}_\Gamma(p, 0) \longrightarrow \mathrm{Gal}_\Gamma(p', 0)$ on Galois group.
- $(f, 1_B)\colon p \longrightarrow p$ induces identity on Galois group.

Properties of Galois group functor

Morphisms in $\mathsf{NExt}_\Gamma(\mathscr{C})$

$$\begin{array}{ccc} E & \overset{f}{\underset{g}{\rightrightarrows}} & E' \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle p'} \\ B & \underset{b}{\longrightarrow} & B' \end{array}$$

induce

- homotopy on kernel pairs and Galois groupoids,
- same morphism $\mathsf{Gal}_\Gamma(p,0) \longrightarrow \mathsf{Gal}_\Gamma(p',0)$ on Galois group.
- $(f, 1_B)\colon p \longrightarrow p$ induces identity on Galois group.

Properties of Galois group functor

Morphisms in $\mathsf{NExt}_\Gamma(\mathscr{C})$

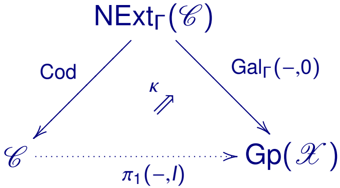
$$\begin{array}{ccc} E & \overset{f}{\underset{g}{\rightrightarrows}} & E' \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle p'} \\ B & \underset{b}{\longrightarrow} & B' \end{array}$$

induce

- homotopy on kernel pairs and Galois groupoids,
- same morphism $\mathsf{Gal}_\Gamma(p,0) \longrightarrow \mathsf{Gal}_\Gamma(p',0)$ on Galois group.
- $(f, 1_B)\colon p \longrightarrow p$ induces identity on Galois group.

Fundamental group functor

Now assume that weakly universal normal extensions exist.



Definition (Janelidze)

Given $B \in \mathscr{C}$, pick weakly universal normal extension $u\colon U \longrightarrow B$, and let

$$\pi_1(B, I) = \mathsf{Gal}_\Gamma(u, 0).$$

Functorial in B because of induced homotopies.

Examples

- Groups and abelian groups: get $\pi_1(B, I) = \mathrm{H}_2(B, \mathbb{Z})$ (group homology).
- Topological example: get usual fundamental group.

Examples

- Groups and abelian groups: get $\pi_1(B, I) = \mathrm{H}_2(B, \mathbb{Z})$ (group homology).
- Topological example: get usual fundamental group.

Natural transformation κ

$$\kappa\colon \pi_1(-, I) \circ \mathrm{Cod} \Longrightarrow \mathrm{Gal}_\Gamma(-, 0)$$

has components

$$\kappa_p\colon \pi_1(B, I) = \mathrm{Gal}_\Gamma(u, 0) \longrightarrow \mathrm{Gal}_\Gamma(p, 0)$$

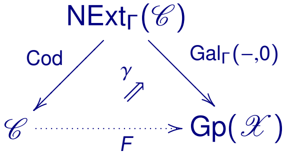
for normal extension $p\colon E \longrightarrow B$, induced by (any)

$$\begin{array}{ccc} U & \overset{h}{\dashrightarrow} & E \\ {\scriptstyle u}\downarrow & & \downarrow{\scriptstyle p} \\ B & = & B \end{array}$$

That is, $\kappa_p = \mathrm{Gal}_\Gamma((h, 1_B), 0)$.

Universality of κ

Given

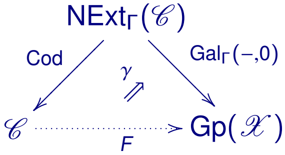


define $\alpha\colon F \Longrightarrow \pi_1(-, I)$ by $\alpha_B = \gamma_u\colon FB \longrightarrow \pi_1(B, I)$.
Then

- α is natural by naturality of γ;
- $\kappa_p \circ \alpha_{\mathrm{Cod}\, p} = \gamma_p$ for all normal extensions p, by naturality of γ;
- α is unique: given β with $\kappa_p \circ \beta_{\mathrm{Cod}\, p} = \gamma_p$ for all normal p, get $\alpha_B = \beta_B$ as κ_u is an iso for weakly universal u.

Universality of κ

Given

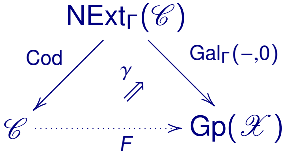


define $\alpha\colon F \Longrightarrow \pi_1(-, I)$ by $\alpha_B = \gamma_u\colon FB \longrightarrow \pi_1(B, I)$.
Then

- α is natural by naturality of γ;
- $\kappa_p \circ \alpha_{\mathsf{Cod}\,p} = \gamma_p$ for all normal extensions p, by naturality of γ;
- α is unique: given β with $\kappa_p \circ \beta_{\mathsf{Cod}\,p} = \gamma_p$ for all normal p, get $\alpha_B = \beta_B$ as κ_u is an iso for weakly universal u.

Universality of κ

Given

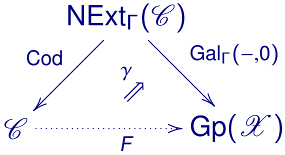


define $\alpha\colon F \Longrightarrow \pi_1(-, I)$ by $\alpha_B = \gamma_u\colon FB \longrightarrow \pi_1(B, I)$.
Then

- α is natural by naturality of γ;
- $\kappa_p \circ \alpha_{\mathsf{Cod}\, p} = \gamma_p$ for all normal extensions p, by naturality of γ;
- α is unique: given β with $\kappa_p \circ \beta_{\mathsf{Cod}\, p} = \gamma_p$ for all normal p, get $\alpha_B = \beta_B$ as κ_u is an iso for weakly universal u.

Universality of κ

Given

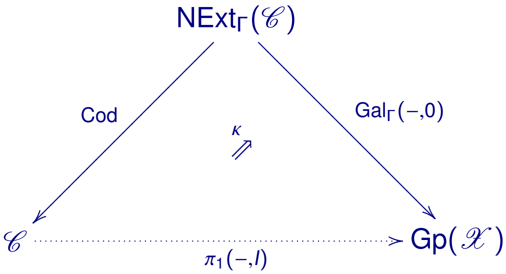


define $\alpha\colon F \Longrightarrow \pi_1(-,I)$ by $\alpha_B = \gamma_u\colon FB \longrightarrow \pi_1(B,I)$.
Then

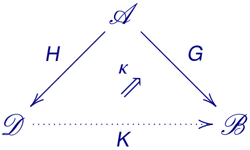
- α is natural by naturality of γ;
- $\kappa_p \circ \alpha_{\mathsf{Cod}\,p} = \gamma_p$ for all normal extensions p, by naturality of γ;
- α is unique: given β with $\kappa_p \circ \beta_{\mathsf{Cod}\,p} = \gamma_p$ for all normal p, get $\alpha_B = \beta_B$ as κ_u is an iso for weakly universal u.

Kan extension

So indeed we have a Kan extension

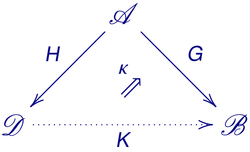


What we used



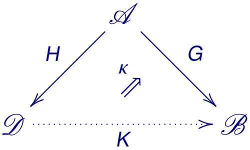
- $H(f) = H(g) \Rightarrow G(f) = G(g)$
- for all $D \in \mathscr{D}$ there is $U \in \mathscr{A}$ with $H(U) = D$ and for all $A \in \mathscr{A}$, $\mathrm{Hom}_{\mathscr{A}}(U, A) \longrightarrow \mathrm{Hom}_{\mathscr{D}}(D, HA)$ is surjective.
- Define $K(D) = G(U)$, well-defined and functorial by above properties.
- Get Kan-extension.

What we used



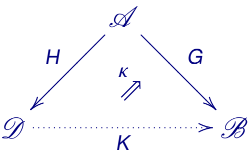
- $H(f) = H(g) \Rightarrow G(f) = G(g)$
- for all $D \in \mathscr{D}$ there is $U \in \mathscr{A}$ with $H(U) = D$ and for all $A \in \mathscr{A}$, $\mathrm{Hom}_{\mathscr{A}}(U, A) \longrightarrow \mathrm{Hom}_{\mathscr{D}}(D, HA)$ is surjective.
- Define $K(D) = G(U)$, well-defined and functorial by above properties.
- Get Kan-extension.

What we used



- $H(f) = H(g) \Rightarrow G(f) = G(g)$
- for all $D \in \mathscr{D}$ there is $U \in \mathscr{A}$ with $H(U) = D$ and for all $A \in \mathscr{A}$, $\mathrm{Hom}_{\mathscr{A}}(U, A) \longrightarrow \mathrm{Hom}_{\mathscr{D}}(D, HA)$ is surjective.
- Define $K(D) = G(U)$, well-defined and functorial by above properties.
- Get Kan-extension.

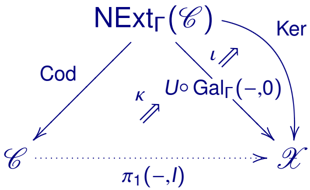
What we used



- $H(f) = H(g) \Rightarrow G(f) = G(g)$
- for all $D \in \mathscr{D}$ there is $U \in \mathscr{A}$ with $H(U) = D$ and for all $A \in \mathscr{A}$, $\mathrm{Hom}_{\mathscr{A}}(U, A) \longrightarrow \mathrm{Hom}_{\mathscr{D}}(D, HA)$ is surjective.
- Define $K(D) = G(U)$, well-defined and functorial by above properties.
- Get Kan-extension.

Second Kan extension

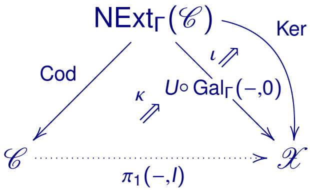
Get also Kan extension



- For $p\colon E \longrightarrow B$ normal, $U\circ\mathrm{Gal}_\Gamma(p,0) = \mathrm{Ker}\,p \cap \mathrm{Ker}\,\eta_E$.
- So $\iota_p\colon \mathrm{Ker}\,p \cap \mathrm{Ker}\,\eta_E \longrightarrow \mathrm{Ker}\,p$ is a mono.
- Any natural transformation $F\circ\mathrm{Cod} \Longrightarrow \mathrm{Ker}$ factors over $U\circ\mathrm{Gal}_\Gamma(-,0)$.
- So the outer diagram is also a Kan extension.

Second Kan extension

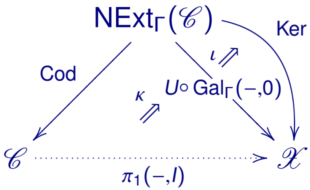
Get also Kan extension



- For $p\colon E \longrightarrow B$ normal, $U\circ \mathsf{Gal}_\Gamma(p,0) = \mathsf{Ker}\, p \cap \mathsf{Ker}\, \eta_E$.
- So $\iota_p\colon \mathsf{Ker}\, p \cap \mathsf{Ker}\, \eta_E \longrightarrow \mathsf{Ker}\, p$ is a mono.
- Any natural transformation $F\circ \mathsf{Cod} \Longrightarrow \mathsf{Ker}$ factors over $U\circ \mathsf{Gal}_\Gamma(-,0)$.
- So the outer diagram is also a Kan extension.

Second Kan extension

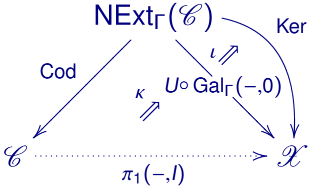
Get also Kan extension



- For $p\colon E \longrightarrow B$ normal, $U\circ \mathrm{Gal}_\Gamma(p,0) = \mathrm{Ker}\, p \cap \mathrm{Ker}\, \eta_E$.
- So $\iota_p\colon \mathrm{Ker}\, p \cap \mathrm{Ker}\, \eta_E \longrightarrow \mathrm{Ker}\, p$ is a mono.
- Any natural transformation $F\circ \mathrm{Cod} \Longrightarrow \mathrm{Ker}$ factors over $U\circ \mathrm{Gal}_\Gamma(-,0)$.
- So the outer diagram is also a Kan extension.

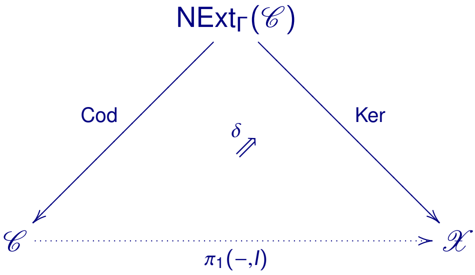
Second Kan extension

Get also Kan extension



- For $p\colon E \longrightarrow B$ normal, $U \circ \mathsf{Gal}_\Gamma(p,0) = \operatorname{Ker} p \cap \operatorname{Ker} \eta_E$.
- So $\iota_p\colon \operatorname{Ker} p \cap \operatorname{Ker} \eta_E \longrightarrow \operatorname{Ker} p$ is a mono.
- Any natural transformation $F \circ \mathsf{Cod} \Longrightarrow \mathsf{Ker}$ factors over $U \circ \mathsf{Gal}_\Gamma(-,0)$.
- So the outer diagram is also a Kan extension.

Second Kan extension



Topological example

- Make it a pointed example by considering basepoints.
- Not every locally connected topological space has weakly universal normal extension.
- Restrict: connected, locally path connected, semi-locally simply connected spaces have *universal* normal extension.
- Proofs still work for restricted settings.

Topological example

- Make it a pointed example by considering basepoints.
- Not every locally connected topological space has weakly universal normal extension.
- Restrict: connected, locally path connected, semi-locally simply connected spaces have *universal* normal extension.
- Proofs still work for restricted settings.

Topological example

- Make it a pointed example by considering basepoints.
- Not every locally connected topological space has weakly universal normal extension.
- Restrict: connected, locally path connected, semi-locally simply connected spaces have *universal* normal extension.
- Proofs still work for restricted settings.

Topological example

- Make it a pointed example by considering basepoints.
- Not every locally connected topological space has weakly universal normal extension.
- Restrict: connected, locally path connected, semi-locally simply connected spaces have *universal* normal extension.
- Proofs still work for restricted settings.

Connecting homomorphism

Given fibration $(F, x) \longrightarrow (E, x) \longrightarrow (B, y)$, with F the fibre over y, universal cover (U, w) of B gives

$$\begin{array}{ccc} (U,w) & & \\ {\scriptstyle e}\downarrow & \searrow^{u} & \\ (E,x) & \xrightarrow[p]{} & (B,y) \end{array} \qquad \text{restricting to} \qquad \begin{array}{ccc} (U,w) & & \\ {\scriptstyle e'}\downarrow & \searrow^{u} & \\ (E_x,x) & \xrightarrow[p']{} & (B,y) \end{array}$$

where E_x is connected component of x.

Connecting homomorphism

This gives rise to SES

$$0 \longrightarrow \pi_1(E,x) \longrightarrow \pi_1(B,y) \longrightarrow (F \cap E_x, x) \longrightarrow 0$$

and

$$0 \longrightarrow (F \cap E_x, x) \longrightarrow (F,x) \longrightarrow \pi_0(E,x) \longrightarrow 0$$

which paste to usual homotopy sequence of the fibration:

$$0 \longrightarrow \pi_1(E,x) \longrightarrow \pi_1(B,y) \longrightarrow \pi_0(F,x) \longrightarrow \pi_0(E,x) \longrightarrow 0$$

Here the connecting homomorphism is component of δ from the Kan extension!

Connecting homomorphism

This gives rise to SES

$$0 \longrightarrow \pi_1(E,x) \longrightarrow \pi_1(B,y) \longrightarrow (F \cap E_x, x) \longrightarrow 0$$

and

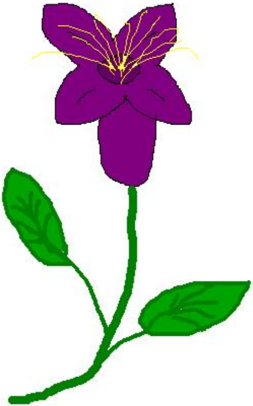
$$0 \longrightarrow (F \cap E_x, x) \longrightarrow (F,x) \longrightarrow \pi_0(E,x) \longrightarrow 0$$

which paste to usual homotopy sequence of the fibration:

$$0 \longrightarrow \pi_1(E,x) \longrightarrow \pi_1(B,y) \longrightarrow \pi_0(F,x) \longrightarrow \pi_0(E,x) \longrightarrow 0$$

Here the connecting homomorphism is component of δ from the Kan extension!

Thanks for listening!



References

- T. Everaert, J. Goedecke, and T. Van der Linden, *The fundamental group functor as a Kan extension*, submitted, 2013.
- F. Borceux and G. Janelidze, *Galois theories*, Cambridge Stud. Adv. Math., vol. 72, Cambridge Univ. Press, 2001.
- J. Goedecke and T. Van der Linden, *On satellites in semi-abelian categories: Homology without projectives*, Math. Proc. Cambridge Philos. Soc. **147** (2009), no. 3, 629–657.
- G. Janelidze, *Pure Galois theory in categories*, J. Algebra **132** (1990), no. 2, 270–286.
- G. Janelidze, *Galois groups, abstract commutators and Hopf formula*, Appl. Categ. Structures **16** (2008), 653–668.