PSSL

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Abstracts

Igor Baković: Fibrations in tricategories

The main purpose of the talk is to introduce a notion of a *fibration in tricategories* which generalizes a notion of a fibration in a 2-category introduced by Johnstone [2]. *Tricategories* were introduced by Gordon, Power and Street [3] as the weakest possible generalization of bicategories, on the 3-dimensional level. It was already pointed by Street that fibrations of bicategories are quite different from *fibrations in bicategories* [4]. The main distinction between fibrations of bicategories, introduced by the author [1] and fibrations in bicategories is that the latter should be seen as fibrations in the tricategory $mathrmBicat_s$ of bicategories, strict homomorphisms, pseudonatural transformations and modifications. Our definition of a fibration in tricategories is in one sense more general then Johnstone's notion, (by the shift in dimension) and in the other sense it is more stricter (since it defines only those fibrations which are strict in a suitable sense). This machinery provides a workable tool mainly due to the fact that the tricategory Bicat_s has an underlying category of bicategories and strict homomorphisms which is finitely complete and therefore has all pullbacks.

References:
[1] I. Baković, Fibrations of bicategories, preprint
[2] P.T. Johnstone, Fibrations and partial products in a 2-category, Applied Categorical Structures, Vol. 1, Nr. 2 (1993) , p. 141-179.
[3] R. Gordon, A. J. Power, and R. Street, Coherence for tricategories, Mem. Amer. Math. Soc. 117 (1995), no. 558, vi+81 pp
[4] R. Street, Fibrations in bicategories, Cahiers Topologie Géom. Différentielle 21 (1980), no. 2, 111-160.

Marie Bjerrum: The closure of a class of finite limits by mixed interchange in Set

It is a well known fact, since the beginning of category theory, that filtered colimits commute with finite limits in Set. This classical result and its consequences generalises to a few other cases (e.g finite products commute with sifted colimits in Set), but so far it has not been determined exactly which classes of limits and colimits commute, even in Set. We present this question, of mixed interchange, as a Galois correspondence between classes of limit indices and classes of colimit indices and give, for a finite limit index $\mathbb{I}$, necessary and sufficient conditions on a small colimit index $\mathbb{J}$, such that $\mathbb{J}$-colimits commute with $\mathbb{I}$-limits in Set. As a consequence we obtain all fixed points, of this Galois correspondence, parting from classes of finite limit indices, i.e. we determine the closure of a class of finite limits, by mixed interchange in Set.

John Bourke: 2-dimensional monadicity

It has been known since [1] that the varieties of 2-dimensional universal algebra exhibit behaviour which cannot be captured using 2-category theory alone, but require the ability to speak of the subcollection of strict morphisms. A simple example is that the 2-category of monoidal categories and lax monoidal functors has products, formed as in Cat, with moreover the product projections "strict monoidal" and jointly detecting "strictness". In seeking to understand more complex phenomena of a similar flavour the authors of [2] were led to introduce the notion of an F-category, which is a 2-category with a specified subcollection of "tight" morphisms. For example there is an F-category of monoidal categories and lax monoidal functors, with tight morphisms the strict ones, and now the above property can be described as a limit lifting property of the forgetful F-functor from there to Cat.
In the present talk I will describe a series of properties of such forgetful F-functors to 2-categories, whereby any F-functor satisfying them must be the forgetful F-functor from an F-category of algebras for a 2-monad, souping up Beck's theorem from the strict world to cover each weaker kind of morphism.

References:
[1] R. Blackwell, G.M. Kelly and A.J. Power. Two-Dimensional Monad Theory. J. Pure Appl. Algebra 59 (1989), 1-41.
[2] S. Lack and M. Shulman. Enhanced 2-categories and limits for lax morphisms, Advances in Mathematics 229 (2011), 294-256.

Eugenia Cheng: Multivariable adjunctions and mates

In this talk I will present the notion of "cyclic double multicategory", as a structure in which to organise multivariable adjunctions and mates. The most common example of a 2-variable adjunction is the hom/tensor/cotensor trio of functors; we generalise this situation to $n+1$ functors of n variables. Furthermore, we generalise the mates correspondence, which enables us neatly to pass between natural transformations involving left adjoints and those involving right adjoints. While the standard mates correspondence is elegantly described using an isomorphism of double categories, the multivariable version needs the framework of "double multicategories". Moreover, we show that the analogous isomorphisms of double multicategories give a cyclic action on the multimaps, yielding the notion of "cyclic double multicategory".
This is joint work with Nick Gurski and Emily Riehl, and is motivated by and applied to Riehl's approach to algebraic monoidal model categories.

Dion Coumans: Generalizing canonical extension to the categorical setting

In the 1950s Jonsson and Tarksi introduced the notion of canonical extension for Boolean algebras with operators. In this setting, canonical extension provides an algebraic description of Stone's topological duality. By now, the theory of canonical extensions has been developed further and it has proven be a powerful tool in the algebraic study of propositional logics. After a brief introduction in this theory, we'll define a notion of canonical extension for coherent categories, the categorical analogues of distributive lattices. This notion extends the existing notion of canonical extension for distributive lattices (viewed as coherent categories). Furthermore, it may be characterized by a universal property. This construction opens the way to applications of the theory of canonical extension in the study of first order logics.

Our construction of canonical extension for coherent categories has led to an alternative description of the topos of types, introduced by Makkai in 1981. This allows us to give new and transparent proofs of some properties of the action of the topos of types construction on morphisms. Furthermore, we apply this description to relate, for a coherent category, its topos of types to its category of models (in **Set**). If time permits, we shortly elaborate on these results.

Marino Gran: Weighted commutators in semi-abelian categories

A new notion of "weighted centrality" of subobjects is introduced in the context of semi-abelian categories [1]. Both the Huq commutator of subobjects [2] and the categorical commutator of equivalence relations [3] can be seen as special cases of the general commutator corresponding to the "weighted centrality", that we call the "weighted commutator". The relationship with some research in universal algebra [4] will also be considered. This work is in collaboration with George Janelidze and Aldo Ursini.

References: [1] G. Janelidze, L. Marki and W. Tholen, Semi-abelian categories, J. Pure Appl. Algebra 168, 367-386 (2002)

[2] S. A. Huq, Commutator, nilpotency and solvability in categories, Quart. J. Math. Oxford 19, no. 2, 363-389 (1968)

[3] M.C. Pedicchio, A categorical approach to commutator theory, J. Algebra 177, 647-657 (1995)

[4] A. Ursini, On subtractive varieties I, Algebra Univers. 31, 204-222 (1994)

Nick Gurski: The Gray tensor product via factorization

It is a folklore result that the Gray tensor product can be obtained by factoring the canonical map from the funny tensor product of 2-categories to the Cartesian product. I will discuss how this result can be explained by combining two independently interesting lines of investigation: first how to obtain factorization systems on V-Cat, and second some basic coherence results related to coherence for tricategories.

Chris Heunen: Relative Frobenius algebras are groupoids

We functorially characterize groupoids as special dagger Frobenius algebras in the category of sets and relations. This is then generalized to a non-unital setting, by establishing an adjunction between H^*-algebras in the category of sets and relations, and locally cancellative regular semigroupoids. Finally, we study a universal passage from the former setting to the latter.

Jürgen Koslowksi: A categorical approach to generating online machine models for formal languages from grammars

Consider the main classes of formal languages in the Chomsky hierarchy (types 3 – 0). The machine models usually employed to characterize these classes, namely finite automata (FAs), push-down automata (PDAs), linear bounded automata (LBAs), and Turing machines (TMs), respectively, and their modes of operation show less of a family resemblance than their defining grammars. In particular, FAs and PDAs can be viewed as online algorithms, that do not need to know their whole input in advance, whereas LBSs and TMs correspond to offline algorithms. While the more complicated machines of course can simulate the simpler ones, this involves a fair bit of technical detail.

We continue the program started in Vancouver 2011 to use a categorical approach to grammars for deriving machine models in the same categorical framework. This is based on Bob Walters' alternative presentation of context-free grammars by means of (co)multi graphs and utilizes a form of (un)currying to connect grammars in various normal forms. Just as one obtains "pure" PDAs (with just one external state) from context-free grammars in Greibach normal form by insisting on left derivations (thereby constraining access to the stack), one can proceed in the other cases as well. For the regular case this recovers the familiar connection between FAs and type 3 grammars (or rather their analogues from a Walters-type point of view). On the other hand, type 0 grammars essentially yield "pure" 2-PDAs as an online alternative to Turing machines.

While this shows that external states are not essential for the power of the machine models, they certainly are convenient and relevant to the issue of determinacy. Hence we also show how to incorporate external states into the categorical model.

Literature:

[1] R. F. C. Walters: A note on context-free languages, Journal of Pure and Applied Algebra 62 (1989), 199–203

Finn Lawler: Biprofunctors and proarrow equipments

Proarrow equipments, originally defined by Wood, are bicategorical structures intended as universes in which to do formal category theory. There are several other definitions besides Wood's, notably Verity's and Shulman's. We will define a tricategory BiProf of bicategories, profunctors (i.e. modules/distributors), and higher morphisms, in which proarrow equipments can be seen as certain (pseudo)monads. The essential link between the different definitions is given by the construction of Kleisli objects in BiProf, which we will explain.

The obvious embedding $\mathrm{Bicat} \to \mathrm{BiProf}$ satisfies a 3-categorical version of the axioms for an equipment, and we will examine to what extent equipments themselves, and their morphisms, can be seen as the result of a 'monads-and-modules' construction applied to this '3-equipment'.

Tom Leinster: The eventual image

Every endomorphism of a finite set gives rise canonically to an idempotent endomorphism of that set. The same is true for finite-dimensional vector spaces and compact metric spaces. I will explain all three scenarios, and state a unifying categorical theorem. The image of the idempotent is the so-called "eventual image" of the original endomorphism f, that is, the intersection of the images of all the iterates f^n. The eventual image has not one but two universal properties, dual to one another.

Nelson Martins-Ferreira: Topological space objects

A topological space can be defined in several equivalent ways: open sets; closed sets; closure operators; neighbourhood systems; etc. In spite of that, there is not yet a well established concept for a internal topological space object.

In this talk we will explore the fact that to each topological space (B, τ) we can associate a projection map $p\colon E \to B$, with $E = \{(x, U) | x \in U \in \tau\}$, and a section $s\colon B \to E$, given by $s(x) = (x, B)$ in E. This fact suggests to consider an internal topological space object as a span $L(B) \xleftarrow{N} E \xrightarrow{p} B$, where L is a covariant endofunctor, with the property that (N, p) is a jointly monic pair of morphisms, there exists a section $s\colon B \to E$, and satisfying some more suitable axioms that will be given during the talk. The final objective is to obtain a definition for an internal object such that in the category Set of sets and maps it gives a topological space, while for instance in the category Grp of groups it gives a topological group. This is still work in progress so that any comments and suggestions are welcome.

Daniela Petrisan: Relation lifting, with an application to the many-valued cover modality

-joint work with Jiri Velebil, Alexander Kurz and Marta Bilkova-
We introduce basic notions and results about relation liftings on categories enriched in a commutative quantale. We derive two necessary and sufficient conditions for a 2-functor T to admit a functorial relation lifting: one is the existence of a distributive law of T over the "powerset monad" on categories, one is the preservation by T of "exactness" of certain squares. Both characterizations are generalizations of the "classical" results known for set functors: the first characterization generalizes the existence of a distributive law over the genuine powerset monad, the second generalizes preservation of weak pullbacks.
These results enable us to compute predicate liftings of endofunctors of, for example, generalized (ultra)metric spaces. As an example, we give a novel definition of a generalized power functor on categories enriched in commutative quantales. Under certain assumption on the quantale, this functor admits a relation lifting and allows us to study the coalgebraic cover modality in the enriched setting.

Jiri Rosicky: Rigidification of algebras over essentially algebraic theories

Badzioch and Bergner proved a rigidification theorem saying that each simplicial homotopy algebra is weakly equivalent to a simplicial algebra. The question is whether this result can be extended from algebraic theories to finite limit theories and from simplicial sets to more general monoidal model categories. We will present some answers to this question.

Michael Shulman: Euler characteristics of colimits

In some cases, the "size" (cardinality, dimension, or Euler characteristic) of a colimit can be calculated from the sizes of the objects in the diagram. We prove that if "size" means the trace of the identity morphism of a dualizable object, then there is such a formula for any "absolute colimit" (one preserved by all enriched functors). This includes additivity for direct sums, the inclusion-exclusion formula for stable homotopy pushouts, and "Burnside's lemma" for finite group actions. The proof uses the machinery of bicategorical traces, and applies more generally to traces of induced endomorphisms of absolute colimits. This is joint work with Kate Ponto.

Mike Stay: Compact closed bicategories

I conjecture that there exists a compact closed 2-theory $\mathrm{Th}(\mathrm{CartClosed})$ of a cartesian closed category such that the 2-category $\hom(\mathrm{Th}(\mathrm{CartClosed}), \mathrm{Prof})$ of sylleptic monoidal functors, braided monoidal transformations, and monoidal modifications is 2-

equivalent to the 2-category of cartesian closed categories, braided monoidal closed functors, and monoidal closed natural isomorphisms.

Paul Taylor: Semilattice bases for Locally Compact Spaces

A semilattice basis for a locally compact space has a formal binary intersection operation and cover or way-below relation. The axioms for these are given, together with the construction of the space. The points are rounded, bounded, located filters, very similar to Dedekind cuts. A simple modification to the cover relation that disables its notion of locatedness yields a generalisation of the Interval Domain. This leads into work that was presented at Domains X on computation in locally compact spaces.

Benno van den Berg: Predicative toposes revisited

The internal logic of a topos is constructive in that only the laws of intuitionistic logic are generally valid; but it is not constructive from the point of view of Martin-Loef type theory or Aczel's CZF, because its internal logic is essentially impredicative. In response, people have suggested there ought to be a notion of a "predicative topos": a topos-like structure whose internal logic would also be constructive from the type-theoretic or CZF point of view. In my PhD thesis I investigated several possibilities (all essentially due to Moerdijk and Palmgren), without reaching a definite conclusion. In this talk I will put forward a new definition of a predicative topos and explain why I think it has all the right properties.

Tim Van der Linden: The Commutator Condition for higher central extensions

We prove that all semi-abelian categories with the the *Smith is Huq* property satisfy the *Commutator Condition (CC)*: higher central extensions may be characterised in terms of binary (Huq or Smith) commutators. In fact, even binary Higgins commutators suffice. As a consequence, in the presence of enough projectives we obtain explicit Hopf formulae for homology with coefficients in the abelianisation functor, and an interpretation of cohomology with coefficients in an abelian object in terms of equivalence classes of higher central extensions. We also give a counterexample against (CC) in the semi-abelian category of loops.

Jamie Vicary: A 2-Categorical Formalism for Quantum Information

I describe way to encode quantum protocols as diagrams in a monoidal 2-category. To reproduce conventional quantum mechanics, the correct target is $2\mathrm{Hilb}$, the 2-category of 2-Hilbert spaces. The graphical calculus for 2-categories allows quantum algorithms to be represented as topological structures, which gives insight into their structure and allows the complete space of quantum algorithms to be examined in a novel way.

Marek Zawadowski: The Theory of Analytic monads

(joint work with Stanislaw Szawiel)
The categories of equational theories, Lawvere theories, and finitary monads on Set are equivalent and provide three different ways of describing categories of algebras in Set. The categories of symmetric operads and of those with non-standard amalgamations also provide descriptions of categories of algebras but of a more specific kind. In [Z] it was shown that analytic monads on Set and symmetric operads describe the same categories of algebras. Similarly, (finitary) polynomial monads on Set and operads with non-standard amalgamations describe the same categories of algebras.
In this talk I will describe the categories of Lawvere theories and equational theories that correspond to such categories of algebras. In case of Lawvere theories the characterization is expressed in terms of factorization systems. The characterization of equational theories corresponding to polynomial monads corrects a statement from [CJ].
If time permits I will also characterize the category of Lawvere theories and the category of monads that correspond to the category of regular equational theories.

References:
[CJ] A. Carboni, P. T. Johnstone, Connected limits, familial representability, and Artion, glueing, Mathematical Structures in Comp. Science (1995), vol 5, pp. 441-459.
Z] M. Zawadowski, Lax Monoidal Fibrations, in Models, Logics, and Higher-Dimensional Categories: A Tribute to the Work of Mihály Makkai (Bradd Hart, Thomas G. Kucera, Philip J. Scott, Robert A.G. Seely, editors) (CRM Proceedings 53, 2011), pp. 341-424.