In the proof of "Kleisli is initial", why is $H'(\varepsilon_B) = \varepsilon_{FB}$?

Remember: in any adjunction, $f : FA \longrightarrow B$ corresponds to

$$A \xrightarrow{\eta_A} GFA \xrightarrow{Gf} GB.$$

and $F^T G^T B = TB \xrightarrow{\varepsilon_B} B = 1_{TB}$.

We have $GH' = G_T$ and $H'F_T = F$. (so $H'B = FB$ on objects).

So $H'(\varepsilon_B)$ corresponds under the adjunction to

$$H'(\varepsilon_B) : FTB = FGFB \longrightarrow FB \quad \longleftrightarrow \quad \begin{tikzcd} GFB \arrow[r, "\eta_{GFB}"] & GFGFB \arrow[d, "GH'(\varepsilon_B) = G_T(\varepsilon_B)"] \\ & GFB \end{tikzcd}$$

But $G_T(\varepsilon_B) = TTB \xrightarrow{T1_{TB}} TTB \xrightarrow{\mu_B} TB$

So

$$\begin{tikzcd} GFB \arrow[r, "\eta_{GFB}"] \arrow[ddr, "1_{TB}"'] & GFGFB \arrow[d, "T1_{TB}"] \\ & GFGFB \arrow[d, "\mu_B"] \\ & GFB \end{tikzcd}$$

But $1_{TB} : GFB \longrightarrow GFB$ corresponds to $\varepsilon_{FB} : FB \to FGFB$,

So $H'(\varepsilon_B) = \varepsilon_{FB}$ (as the correspondance is a bijection).