Groups Ia Practice Sheet C

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These questions are not supposed to form the work for one of the regular 4 groups supervisions, but instead they give you opportunities to practise getting used to axioms and definitions in your own time. If you find this useful, try to make similar questions for yourself on later material of the course.

Homomorphisms

1. Show that $f\colon \mathbb{Z} \longrightarrow \mathbb{Z}$ given by $f(n) = 2n$ is a group homomorphism. Similarly show $f_k\colon \mathbb{Z} \longrightarrow \mathbb{Z}$ with $f_k(n) = kn$ is a group homomorphism. Can you think of any others?

2. Show that $f\colon \mathbb{R} \longrightarrow \mathbb{R}^3$ with $f(x) = (x, x, x)$ and $g\colon \mathbb{R}^3 \longrightarrow \mathbb{R}$ with $g((x, y, z)) = x$ are group homomorphisms. What about $h\colon \mathbb{R}^3 \longrightarrow \mathbb{R}$ with $h((x, y, z)) = x + y + z$? Can you find any more similar group homomorphisms?

3. Recall that $\mathbb{Q}^* = \mathbb{Q} \setminus \{0\}$ forms a group under multiplication. Show that $f\colon \mathbb{Q}^* \longrightarrow \mathbb{Q}^*$ with $f(\frac{a}{b}) = \frac{b}{a}$ is a group homomorphism. A similar idea with $g\colon G \longrightarrow G$ defined by $g(a) = a^{-1}$ only works for abelian groups! (Why?)

4. Show that $f\colon \mathbb{Z} \longrightarrow \mathbb{Z}$ with $f(n) = n + 1$ is *not* a group homomorphism.

5. Show that the only constant function $f\colon G \longrightarrow H$ between to groups which is a group homomorphism is the one defined by $f(a) = e$ for all $a \in G$.

6. Let $f\colon G \longrightarrow H$ be a group homomorphism. Show that $f(a^{-1}) = f(a)^{-1}$. [Remember that inverses are unique!]